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A cosmological model from emergence of space

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Abstract

Many studies have been carried out since T.Padmanabhan proposed that the cosmic acceleration can be understood from the perspective that spacetime dynamics is an emergent phenomenon. Motivated by such a new paradigm, we firstly study the de Sitter universe from emergence of space. After that we investigate the universes in general cases and then narrow down our discussions into one of them with a detailed discussion of the possibility in describing our real universe classically. Furthermore, a constraint on Ht and a estimated value of $\tilde{\Omega}_\Lambda$ (caused by ρ_{vac}) can be derived from our model, the comparison with experiments is also presented. The results show the validity of our model.

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I. INTRODUCTION

The discovery of black hole thermodynamics [1, 2] has helped us to know the nature of gravity. With the deep study of the connection between gravitation and thermodynamics, physicists generally believe that the space is emergent which means gravity may not be a fundamental interaction but an emergent phenomenon now.

It was first shown by Jacobson [3] that the Einstein field equations can be derived from the Clausius relation on a local Rindler causal horizon. Verlinde [4], by suggesting that gravity should be explained as an entropic force caused by changes of entropy associated with the information on the holographic screen, put forward a great step towards understanding the nature of gravity. With the holographic principle and the equipartition law of energy, Verlinde derived the Newton's law and the Einstein fields equations in a relativistic regime. Earlier, Padmanabhan [5] observed that the equipartition law of energy for horizon degrees of freedom (DOF), combined with the thermodynamics relation $S = \frac{E}{2T}$, leads to Newton's law of gravity.

In most cases, only the gravitational field is treated as an emergent phenomenon, with the pre-exiting background geometric manifold assumed. A more complete way is to treat spacetime itself as an emergent structure as well, and it was finally proposed by Padmanabhan [6, 7]. He argued that the spatial expansion of our universe is due to the difference between the surface DOF and the bulk DOF in the region of emerged space. Then, he proposed a simple equation $dV/dt = L_P^2 \Delta N$ [6, 7], where V is the Hubble volume and t is the cosmic time. $\Delta N = N_{sur} - N_{bulk}$ with N_{sur} being the number of DOF on the boundary and N_{bulk} being the number in the bulk. Cai [8] generalized the derivation process to the higher (n+1)-dimensional spacetime. He also obtained the Friedmann equations of a flat FRW universe in Gauss-Bonnet and more general Lovelock cosmology by properly modifying the effective volume and the number of DOF on the holographic surface from the entropy formulae of static

spherically symmetric black holes [8]. In Ref [9], on the other hand, the authors generalized the holographic equipartition and derived the Friedmann equations by assuming that (dV/dt) is proportional to a general function $f(\Delta N, N_{sur})$. Note that the authors of [8, 9] only derived the Friedmann equations of the spatially flat FRW universe. In Ref [10], Sheykhi derived the Friedmann equations of the FRW universe with any spatial curvature. The authors of [11] proposed a general equation which can be reduced to the different modified ones in different cases. For more investigations about the novel idea see Refs. [10–15].

In this paper, we use the equation proposed by Padmanabhan to find some characters of the de Sitter universe at first, and then we generalize the important character $\rho + 3p = constant$ to general case. By solving the equation $dV/dt = L_P^2 \Delta N$, we can get the solution of $H(t)$, hence $a(t)$. Therefore we build a cosmological model to study the evolution of $a(t)$ in detail. It is easy to find that our universe would be de Sitter universe far into the future and a constraint on H and t is obtained in our model. Finally, we give a estimated value of $\tilde{\Omega}_\Lambda$(caused by ρ_{vac}) and compare them with the experiments data. This paper is organized as follows: In Section II, a brief review of Padmanabhan's work is presented firstly. In section III, we then discuss a de Sitter universe from emergence of space. In Section IV we present our cosmological model in details and the comparison of our model with experiments. Section V is for conclusions and discussions.

II. EMERGENCE OF SPACE

Padmanabhan [6] noticed that in a pure de Sitter universe with Hubble constant H , the holographic principle can be expressed in terms of

$$N_{sur} = N_{bulk}, \tag{1}$$

where N_{sur} denotes the number of DOF on the spherical surface of Hubble radius H^{-1}, namely $N_{sur} = 4\pi H^{-2}/L_p^2$, with L_p being the Planck length, while the bulk DOF $N_{bulk} = |E|/(1/2)T$. Here $|E| = |\rho + 3p|\, V$, is the Komar energy with the Hubble volume $V = 4\pi/(3H^3)$ and the horizon temperature $T = H/2\pi$. For the pure de Sitter universe, substituting $\rho = -p$ into Eq. (1), the standard result $H^2 = 8\pi L_p^2 \rho/3$ is obtained.

From Eq. (1), one can get $|E| = (1/2)N_{sur}T$, which is the standard equipartition law. Padmanabhan called it *holographic equipartition*, because it relates the effective DOF residing in the bulk to the DOF on the boundary surface. It is known that our real universe is just asymptotically de Sitter. Padmanabhan further suggested that the emergence of space occurs and relates to the difference $\Delta N = N_{sur} - N_{bulk}$. A simple equation was proposed [6]

$$\frac{dV}{dt} = L_p^2 \Delta N. \tag{2}$$

Putting the above definition of each term, one obtains

$$\frac{\ddot{a}}{a} = -\frac{4\pi L_p^2}{3}(\rho + 3p). \tag{3}$$

This is the standard dynamical equation for the FRW universe in general relativity. Using continuity equation $\dot{\rho} + 3H(\rho + p) = 0$, one gets the standard Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{8\pi L_p^2 \rho}{3}, \tag{4}$$

where k is an integration constant, which can be interpreted as the spatial curvature of the FRW universe. Here, Padmanabhan takes $(\rho+3p) < 0$, which makes sense only in the accelerating phase. It means that in order to have the asymptotic holographic equipartition, the existence of dark energy is necessary.

III. DE SITTER UNIVERSE FROM EMERGENCE OF SPACE

As Padmanabhan said, this new idea provides a new paradigm for cosmology. Hence we would like to push an investigation on cosmology from the emergence of space. First, we will begin with the de Sitter universe.

In this case, the Eq. (2) has the form

$$\frac{dV}{dt} = L_p^2(N_{sur} - N_{bulk}) = 0. \tag{5}$$

One can easily obtain

$$\frac{dV}{dt} = \frac{d(\frac{4\pi}{3H^3})}{dt} = 0, \tag{6}$$

which means H is a constant, and then T, V, N_{sur}, N_{bulk} and $|E|$ are all constants respectively. Using $|E| = |\rho + 3p|V$, one can easily have

$$|\rho + 3p| = \frac{3H^2}{4\pi L_p^2} = constant \tag{7}$$

in de Sitter universe. In other words, once H is a constant, what we can have is $|\rho + 3p| = constant$ according to emergence of space and $\rho = -p$ is just one special kind of de Sitter universe. So it is natural to generalize $p = -\rho$ to the general equation of state (EOS) $p = \omega\rho$ (ω can be time dependent).

If we take the accelerating phase $\rho + 3p < 0$. then Eq. (7) would be the term

$$\rho + 3p = -\frac{3H^2}{4\pi L_p^2} = constant = -B_1, \tag{8}$$

In de Sitter universe, combining Eq. (8) and continuity equation $\dot{\rho} + 3H(\rho + p) = 0$, one can have solutions of ρ and p

$$\rho = \begin{cases} \frac{1}{2}a^{-2} + \frac{B_1}{2}, & -1 < \omega < -1/3 \ (\dot{\rho} < 0) \\ \frac{B_1}{2}, & \omega = -1 \ (\dot{\rho} = 0) \\ \frac{B_1}{2} - \frac{1}{2}a^{-2}, & \omega < -1 \ (\dot{\rho} > 0) \end{cases}, \tag{9}$$

$$p = \begin{cases} -\frac{1}{6}a^{-2} - \frac{B_1}{2}, & -1 < \omega < -1/3 \ (\dot{\rho} < 0) \\ -\frac{B_1}{2}, & \omega = -1 \ (\dot{\rho} = 0) \\ -\frac{B_1}{2} + \frac{1}{6}a^{-2}, & \omega < -1 \ (\dot{\rho} > 0) \end{cases}, \tag{10}$$

where $a = Ae^{Ht}$ has absorbed the integral constant. From Eq. (9) and Eq. (10), we can see that there are strong constraints on ρ and ω in de Sitter universe. Such results motivate us to move on to a cosmological model in a more general case.

IV. A COSMOLOGICAL MODEL FROM EMERGENCE OF SPACE

As well known, our real universe will evolve asymptotically to the de Sitter which satisfies the condition of $\rho + 3p = constant$ as demonstrated in Section III. This naturally moves us to wonder whether the de-Sitter universe is the only case under this condition or other possibilities might exist and, if they do, what will they behave like. In other words, does the equation

$$\rho + 3p = \text{constant} = -B_2 \tag{11}$$

only apply to de Sitter universe? Apparently not, Eq. (11) can be obtained in the universe whose

$$\rho = \begin{cases} \frac{1}{2}a^{-2} + \frac{B_2}{2}, & -1 < \omega < -1/3 \ (\dot{\rho} < 0) \\ \frac{B_2}{2}, & \omega = -1 \ (\dot{\rho} = 0) \\ \frac{B_2}{2} - \frac{1}{2}a^{-2}, & \omega < -1 \ (\dot{\rho} > 0) \end{cases}, \tag{12}$$

$$p = \begin{cases} -\frac{1}{6}a^{-2} - \frac{B_2}{2}, & -1 < \omega < -1/3 \ (\dot{\rho} < 0) \\ -\frac{B_2}{2}, & \omega = -1 \ (\dot{\rho} = 0) \\ -\frac{B_2}{2} + \frac{1}{6}a^{-2}, & \omega < -1 \ (\dot{\rho} > 0) \end{cases}. \tag{13}$$

To investigate the general case of this kind of universe, we go back to solve the Eq. (2) with Eq. (11)

$$\frac{dV}{dt} = \frac{d(\frac{4\pi}{3H^3})}{dt} = -\frac{4\pi}{H^4}\dot{H}, \tag{14}$$

$$L_p^2(N_{sur} - N_{bulk}) = L_p^2(\frac{4\pi}{L_p^2 H^2} - \frac{16\pi^2 B_2}{3H^4}). \tag{15}$$

Combining and arranging the above equations, one can have

$$\alpha^2 - H^2 = \frac{dH}{dt}, \tag{16}$$

where $\alpha = \sqrt{\frac{4\pi B_2 L_p^2}{3}}$. The solutions of Eq. (16) are following($t > 0$):

1): $0 < H < \alpha,\ dH/dt > 0$

$$H = \alpha - \frac{2\alpha}{C_1 e^{2\alpha t} + 1}, \tag{17}$$

where C_1 is an integral constant satisfying $C_1 \geqslant 1$ for the request of $H > 0$.

2): $H = \alpha,\ dH/dt = 0$

$$H = \alpha \tag{18}$$

3): $H > \alpha,\ dH/dt < 0$

$$H = \frac{2\alpha}{D_1 e^{2\alpha t} - 1} + \alpha, \tag{19}$$

where D_1 is an integral constant satisfying $D_1 \geqslant 1$ for the request of $H > \alpha$.

4): $-\alpha < H < 0,\ dH/dt > 0$

$$H = \alpha - \frac{2\alpha}{M_1 e^{2\alpha t} + 1}, \tag{20}$$

where M_1 is an integral constant satisfying $0 < M_1 < e^{-2\alpha t}$ for the request of $-\alpha < H < 0$.

5): $H = -\alpha,\ dH/dt = 0$

$$H = -\alpha \tag{21}$$

6): $H < -\alpha,\ dH/dt < 0$

$$H = \alpha - \frac{2\alpha}{1 - N_1 e^{2\alpha t}}, \tag{22}$$

where N_1 is an integral constant satisfying $0 < N_1 < e^{-2\alpha t}$ for the request of $H < -\alpha$.

It is obvious that the first three solutions represent expansion and the last three represent contraction. Considering the fact our universe is expanding, one should be interested in the first three solutions, so we have a follow-up study.

According to $H = \dot{a}/a$, we have $a(t)$:

1): $0 < H < \alpha$

$$a = \frac{C_1 e^{2\alpha t} + 1}{C_2 e^{\alpha t}}, \tag{23}$$

where C_2 is an integral constant satisfying $C_2 > 0$.

2): $H = \alpha$

$$a = F_1 e^{Ht}, \tag{24}$$

where F_1 is an integral constant satisfying $F_1 > 0$.

3): $H > \alpha$

$$a = \frac{D_1 e^{2\alpha t} - 1}{D_2 e^{\alpha t}}, \tag{25}$$

where D_2 is an integral constant satisfying $D_2 > 0$.

Next, let us push forward with a more detailed analysis on these three cases respectively.

1): $0 < H < \alpha,\ dH/dt > 0$

$$H = \alpha - \frac{2\alpha}{C_1 e^{2\alpha t} + 1}\ , \quad a = \frac{C_1 e^{2\alpha t} + 1}{C_2 e^{\alpha t}}, \tag{26}$$

where $C_1 \geqslant 1$ and $C_2 > 0$. In the limit of $t \to 0$, one can have $a_0 = (C_1 + 1)/C_2 > 0$. This means a universe has an initial nonzero scale factor a_0.

2): $H = \alpha,\ dH/dt = 0$

$$H = \alpha,\ a = F_1 e^{Ht}, \tag{27}$$

where $F_1 > 0$. This is exactly the de Sitter uninerse which we have discussed in Section III.

3): $H > \alpha,\ dH/dt < 0$

$$H = \frac{2\alpha}{D_1 e^{2\alpha t} - 1} + \alpha\ ,\ a = \frac{D_1 e^{2\alpha t} - 1}{D_2 e^{\alpha t}}, \tag{28}$$

where $D_1 \geqslant 1$ and $D_2 > 0$. In the limit of $t \to 0$, $a_0 = (D_1 - 1)/D_2$. If one set $D_1 = 1$, then $a_0 = 0$.

So far, we have got all the universe where Eq. (11) is satisfied generally, and de Sitter universe is just one case of them as we predicted. It is natural to wonder whether our real universe could be one of them.

For our universe, $a_0 = 0$. So, we would like to set $D_1 = 1$ in Eq. (28) and we can have

$$H = \frac{2\alpha}{e^{2\alpha t} - 1} + \alpha, \tag{29}$$

$$a = \frac{e^{2\alpha t} - 1}{D_2 e^{\alpha t}} = A(e^{\alpha t} - \frac{1}{e^{\alpha t}}), \tag{30}$$

where $A = 1/D_2 > 0$. It is easy to find that Eq. (30) describes a universe which is asymptotically de Sitter and our universe is in this case as we generally believe.

There are two problems if Eq. (30) is used to describe our universe. First, Eq. (30) is obtained from the universe which satisfies Eq. (11). It is natural to doubt that our universe satisfies Eq. (11). However, it may be correct considering that our late universe would be de Sitter which satisfies Eq. (11). Second, it can not explain the inflation of early universe. However, just as Padmanabhan [6] said, Eq. (2) needs modifications at early universe. Except these two problems, there is no obvious reason to sweep out the possibility of our real universe depictured by Eq. (30). So,

we will take it as a cosmological model from emergence of space and give a detailed investigation.

Since $t > 0$, $\alpha > 0$ and $H > \alpha$, there should be a constraint in Eq. (29). To find out the constraint, one can multiply t on both sides of Eq. (29) and substitute $x = \alpha t$, then one can have

$$Ht = \frac{2x}{e^{2x} - 1} + x \quad (x > 0). \tag{31}$$

Let

$$y = \frac{2x}{e^{2x} - 1} + x \quad (x > 0).$$

After calculating, one can find

$$y' > 0 \ , \ \lim_{x \to 0} y = 1 \ .$$

So the existence of a solution of $\alpha > 0$ requires a constraint on H and t:

$$tH(t) > 1 \quad \text{or} \quad t > 1/H(t). \tag{32}$$

Eq. (32) is applicable for any $t > 0$ (except for $t \to 0$ which represents the inflation of early universe). Then one can have

$$t_0 H_0 > 1, \tag{33}$$

where t_0 is the present age of our real universe and H_0 is the current value of H.

Since the constraint is only derived in our model and has never appeared in other theories before as far as we know, we would like to compare it with the experiments [16–20] from the Wilkinson Microwave Anisotropy Probe (WMAP) and the Planck Mission. The analysis is shown in Table I.

Table I. An analysis of the data of H_0 and t_0 from the WMAP and the Planck Mission. For obtaining $H_0 t_0$, we have changed km/(Mpc·s) into s^{-1} and Ga into s.

Obsever	Data published	H_0 km/(Mpc·s)	t_0(Ga)	$H_0 t_0$	α km/(Mpc·s)
WMAP	2003	71^{+3}_{-3}	$13.7^{+0.2}_{-0.2}$	0.9393∼1.0667	31.66
WMAP	2006	$73.2^{+3.1}_{-3.2}$	$13.73^{+0.16}_{-0.15}$	0.9727∼1.8044	35.71
WMAP	2008	$70.5^{+1.3}_{-1.3}$	$13.72^{+0.12}_{-0.12}$	0.9629∼1.0168	16.86
WMAP	2010	$70.4^{+1.3}_{-1.3}$	$13.75^{+0.11}_{-0.11}$	0.9644∼1.0168	15.90
Planck	2013	$67.80^{+0.77}_{-0.77}$	$13.798^{+0.037}_{-0.037}$	0.9438∼0.9707	no value

Here, we would like to have an explanation of the value of α in Table I. According to Eq. (29), there exists a value of α in our model once $H_0 t_0 > 1$. And the value of α we have calculated in Table I is the maximum one in each case.

All the experimental data in Table I show that $H_0 t_0 \approx 1$ and some experiments have $H_0 t_0 > 1$. Noticing that the experimental date which $H_0 t_0 < 1$ depart from one by 10^{-2}, our model is fairly valid to describe the real universe.

In fact, our theory may provide a new judgment of $H_0 t_0$ >1 for the experimental data, which is indeed supported by most of the experimental data from WMAP. And the departure from $H_0 t_0$ >1 can be understood well as a uncertainty of current measurement accuracy.

What is more, we might actually calculate the vacuum energy if our model is used to describe the real universe. To see this, let us go back to Eq. (12)

$$\rho = \begin{cases} \frac{1}{2}a^{-2} + \frac{B_2}{2}, & -1 < \omega < -1/3 \ (\dot{\rho} < 0) \\ \frac{B_2}{2}, & \omega = -1 \ (\dot{\rho} = 0) \\ \frac{B_2}{2} - \frac{1}{2}a^{-2}, & \omega < -1 \ (\dot{\rho} > 0) \end{cases}$$

where $B_2 = 3\alpha^2/(4\pi L_p^2)$ and a is given in the form as in Eq. (23)$\sim$(25),(30) respectively. Even though it is impossible to confirm which one ($\dot{\rho} < 0$ or $\dot{\rho} = 0$ or $\dot{\rho} > 0$) belongs to the universe described by Eq.(30), one may find that the ρ of our real universe would be $\dot{\rho} < 0$ by noticing that the energy density for matter ρ_M: $\rho_M \propto a^{-3}$ and the energy density for radiation ρ_R: $\rho_R \propto a^{-4}$. Hence we should choose the first line in the RHS of Eq.(12) to be the possible energy content of our universe.

With the energy content given in the form

$$\rho = \frac{1}{2}a^{-2} + \frac{B_2}{2} \ , \tag{34}$$

it is natural to inquire the possible meaning of $B_2/2$. Noticing that $\lim_{a\to+\infty} \rho = B_2/2 = -p$ is the vacuum energy density(ρ_{vac}) of the pure de Sitter which our real universe would be in the far future, we argue that $B_2/2$ is the ρ_{vac} of our real universe and

$$\rho_{vac} = \frac{B_2}{2} = \frac{3\alpha^2}{8\pi L_p^2}. \tag{35}$$

By dimensional analysis, we have

$$[\frac{3\alpha^2}{8\pi L_p^2}] = \frac{[T]^{-2}}{[L]^2} \ , \ [\rho] = \frac{[M]}{[L]^3} = \frac{[T]^{-2}}{[L]^2} \cdot \frac{[T]^2[M]}{[L]}.$$

We now put back the fundamental constants and get

$$\rho_{vac} = \frac{3\alpha^2}{8\pi L_p^2} \cdot \frac{t_p^2 m_p}{L_p} \quad .$$

Since α can be calculated by experimental data H_0 and t_0 in our theory, we are actually building a new relation of H_0, t_0 and vacuum energy which has never appeared in the literature. Using the α in the Table I, one can finally obtain the range of ρ_{vac} :

$$4.733 \times 10^{-28} \sim 2.397 \times 10^{-27} \ kg/m^3. \tag{36}$$

Comparing with the density of dark energy (about $6.91 \times 10^{-27} kg/m^3$) [16–19], there is a difference between the two though it almost has the same order of our estimated value.

In the standard ΛCDM cosmological model, it is believed that the dark energy is caused by the cosmological constant. Hence it is convenient to compare our theoretical results with the experiment data of Ω_Λ.

The definition of Ω_Λ is

$$\Omega_\Lambda = \Lambda/(3H_0^2) \ , \tag{37}$$

where $\Lambda = 8\pi\rho_{vac}$. Using the ρ_{vac} in our model, one can have

$$\tilde{\Omega}_\Lambda = \alpha^2/H_0^2 \quad . \tag{38}$$

According to the values of α and H_0 in Table I, one can finally get the range of $\tilde{\Omega}_\Lambda$:

$$0.049 \sim 0.260 \qquad . \tag{39}$$

And the experimental data of Ω_Λ from the WMAP and the Planck Mission [16–20] have the range of :

$$0.683 \sim 0.772 \quad .$$

It can be seen that our model predicts the approximate but not exact value of cosmological constant. Still, the cosmological constant derived in our model has the same order of the experimental data. The difference may indicate that there are possibly other sources of dark energy such as the quintessence if our model represents the real universe.

V. CONCLUSIONS AND DISCUSSIONS

To summarize, in this paper, we investigated the novel idea proposed by Padmanabhan [6] that the emergence of space and expansion of the universe are due to the

difference between the number of DOF on the holographic surface and the one in the emerged bulk. It is shown that the Friedmann equation of a flat FRW universe can be derived with the help of continuity equation. Since the emergence of space may provide a completely different paradigm to study cosmology [6], we studied the de Sitter universe from emergence of space, and found that there is a constraint on ρ and p (Eq. (7) and Eq. (8)) which can derive solutions of ρ and p (Eq. (9) Eq.(10)). By considering an arbitrary universe whose ρ and p have the form of Eq.(12) and Eq.(13), we generalized Eq.(8) beyond the de Sitter universe and solved Eq.(2).

Among the solutions we obtained, we found a model which has the possibility to describe our real universe. After detailed analysis of our model, we got three important conclusions:

(1) The universe would be de Sitter in its later period. ($t >> 1/\alpha$).

(2) There is a constraint on H and t : $H(t) \cdot t > 1$, and it is applicable for any $t > 0$ (except for $t \to 0$ which represents the inflation of early universe).

(3) The value of vacuum energy and $\tilde{\Omega}_\Lambda$ can be derived in our model.

We made a comparison of our model with experiments. For conclusion (2), the experimental data show that $H_0 t_0$ ranges from $0.9438 \sim 1.8044$ and our model tends to support the WMAP rather than the Planck. For conclusion (3), our model predicts a positive tiny cosmological constant, which is approximate to the experimental data. The difference may indicate that there are probably other sources contributing to the dark energy if our model represents the real universe.

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